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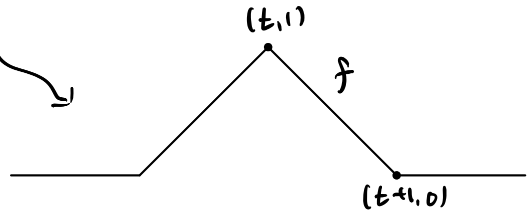
실수 t에 대하여 함수 $f(x)$를

$$f(x)=\begin{cases}1-|x-t| & (|x-t|\le 1)\\ 0 & (|x-t|>1)\end{cases} \longrightarrow [t-1, t+1]$$

이라 할 때, 어떤 홀수 k에 대하여 함수



$$g(t)=\int_k^{k+8} f(x)\cos(\pi x)dx$$

가 다음 조건을 만족시킨다.

함수 $g(t)$가 $t=\alpha$에서 극소이고 $g(\alpha)<0$인 모든 α를 작은 수부터 크기순으로 나열한 것을 $\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_m$ (m은 자연수)라 할 때, $\sum_{i=1}^{m}\alpha_i=45$이다.

$k-\pi^2\sum_{i=1}^{m}g(\alpha_i)$의 값을 구하시오. [4점] 21

sol.)

i) $t+1 \le k$

$$g(t)=\int_k^{k+8} 0\cdot\cos(\pi x)dx = 0$$

ii) $t \le k < t+1$

$$\begin{aligned} g(t) &= \int_k^{k+8} f(x)\cos(\pi x)dx \\ &= \int_k^{t+1}(-x+t+1)\cos(\pi x)dx + \int_{t+1}^{k+8} 0\cdot\cos(\pi x)dx \\ &= t\int_k^{t+1}\cos(\pi x)dx + \int_k^{t+1}(1-x)\cos(\pi x)dx \\ &= \frac{\cos(\pi t)-1}{\pi^2} \end{aligned}$$

iii) $t-1 \le k < t$

$$g(t)=\int_k^{k+8} f(x)\cos(\pi x)\,dx$$
$$=\int_k^{t}(x-t+1)\cos(\pi x)\,dx+\int_t^{t+1}(-x+t+1)\cos(\pi x)\,dx+\int_{t+1}^{k+8}0\cdot\cos(\pi x)\,dx$$
$$=-t\int_k^{t}\cos(\pi x)\,dx+\int_k^{t}(1+x)\cos(\pi x)\,dx+t\int_t^{t+1}\cos(\pi x)\,dx+\int_t^{t+1}(1-x)\cos(\pi x)\,dx$$
$$=\frac{3\cos(\pi t)+1}{\pi^2}$$

iv) $k < t-1$

$$g(t)=\int_k^{k+8} f(x)\cos(\pi x)\,dx$$
$$=\int_k^{t-1}0\cdot\cos(\pi x)\,dx+\int_{t-1}^{t}(x-t+1)\cos(\pi x)\,dx+\int_t^{t+1}(-x+t+1)\cos(\pi x)\,dx+\int_{t+1}^{k+8}0\cdot\cos(\pi x)\,dx$$
$$=\frac{4\cos(\pi t)}{\pi^2}$$

+) t가 계속 증가하다 보면 t가 $[k, k+8]$에 속해지지 않음.

$t=k+4$ 에 대하여 대칭.

$\therefore g(t)=g(2k+8-t)$

$$\therefore g(t)=\begin{cases} 0 & (t\le k-1) \\ \dfrac{\cos(\pi t)-1}{\pi^2} & (k-1<t\le k) \\ \dfrac{3\cos(\pi t)+1}{\pi^2} & (k<t\le k+1) \\ \dfrac{4\cos(\pi t)}{\pi^2} & (k+1<t\le k+7) \\ \dfrac{3\cos(\pi t)+1}{\pi^2} & (k+7<t\le k+8) \\ \dfrac{\cos(\pi t)-1}{\pi^2} & (k+8<t\le k+9) \\ 0 & (t>k+9) \end{cases}$$

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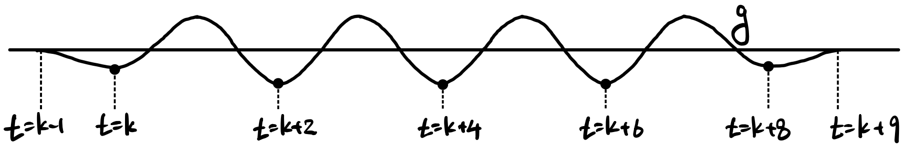
2018학년도 수능(가형) 30번



$\sum^{m} \alpha_i = k + k+2 + k+4 + k+6 + k+8$

$= 5k+20 = 45$

$\therefore k = 5, \ m = 5$

$g(k) = g(k+8) = -\frac{2}{\pi^2}$

$g(k+2) = g(k+4) = g(k+6) = -\frac{4}{\pi^2}$

$\therefore \sum g(\alpha_i) = -\frac{16}{\pi^2}$

$\therefore k - \pi^2 \sum g(\alpha_i) = 21$

sol₂)

$f(x)$의 구간 $[k-1, k+1]$의 길이와 $\cos(\pi x)$의 주기가 같다.

$f(x)$가 실질적으로 t에 따라 달라지기 때문에 상황에 따라 $g(t)$의 함수식도 달라짐.

i) $t+1 \le k$

$[k, k+8]$ 에서 $f(x)=0$

$\therefore g(t) = 0 \longrightarrow g'(t) = 0 \quad (t<k)$

ii) $k-1 < t \le k$

$$g(t) = \int_k^{k+8} f(x)\cos(\pi x)\,dx$$
$$= \int_k^{t+1} (-x+t+1)\cos(\pi x)\,dx + \int_{t+1}^{k+8} 0\cdot\cos(\pi x)\,dx$$
$$= t\int_k^{t+1} \cos(\pi x)\,dx + \int_k^{t+1} (1-x)\cos(\pi x)\,dx$$

$$g'(t) = \int_k^{t+1} \cos(\pi x)\,dx + t\cos(\pi t+\pi) - t\cos(\pi t+\pi)$$
$$= -\frac{1}{\pi}\sin(\pi t) \quad (k-1<t<k)$$

iii) $t-1 \le k < t$

$$g(t) = \int_k^{k+8} f(x)\cos(\pi x)\,dx$$
$$= \int_k^{t} (x-t+1)\cos(\pi x)\,dx + \int_t^{t+1} (-x+t+1)\cos(\pi x)\,dx + \int_{t+1}^{k+8} 0\cdot\cos(\pi x)\,dx$$
$$= -t\int_k^{t} \cos(\pi x)\,dx + \int_k^{t} (1+x)\cos(\pi x)\,dx + t\int_t^{t+1} \cos(\pi x)\,dx + \int_t^{t+1} (1-x)\cos(\pi x)\,dx$$

$$g'(t) = -\int_k^{t} \cos(\pi x)\,dx - t\cos(\pi t) + (t+1)\cos(\pi t) + \int_k^{t+1} \cos(\pi x)\,dx - 2t\cos(\pi t)$$
$$+ t\cos(\pi t) + (t-1)\cos(\pi t)$$

$$= -\frac{3}{\pi}\sin(\pi t) \quad (k<t<k+1)$$

iv) $k < t-1$

$$g(t) = \int_k^{k+8} f(x)\cos(\pi x)\,dx$$
$$= \int_k^{t-1} 0\cdot\cos(\pi x)\,dx + \int_{t-1}^{t} (x-t+1)\cos(\pi x)\,dx + \int_t^{t+1} (-x+t+1)\cos(\pi x)\,dx + \int_{t+1}^{k+8} 0\cdot\cos(\pi x)\,dx$$

$$= -t\int_{t-1}^{t} \cos(\pi x)\,dx + \int_{t-1}^{t} (1+x)\cos(\pi x)\,dx + t\int_t^{t+1} \cos(\pi x)\,dx + \int_t^{t+1} (1-x)\cos(\pi x)\,dx$$

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$$g'(t) = -\int_{t-1}^{t} \cos(\pi x)dx - t\{\cos(\pi t) - \cos(\pi(t-1))\} + (t+1)\cos(\pi t) - t\cos(\pi(t-1))$$

$$+\int_{t}^{t+1} \cos(\pi x)dx + t\{\cos(\pi(t+1)) - \cos(\pi t)\}$$

$$-t\cos(\pi(t+1)) + (t-1)\cos(\pi t)$$

$$= -\frac{4}{\pi}\sin(\pi t) \quad (t > k+1)$$

홀수 k에 대하여 $g(k-1)=0$ 임에 착안하여 $g(t)$가 연속함수가 되도록 적분상수 잡기

+) t가 계속 증가하다 보면 t가 $[k, k+8]$에 속해지지 않음.

$t = k+4$ 에 대하여 대칭.

$\therefore g(t) = g(2k+8-t)$

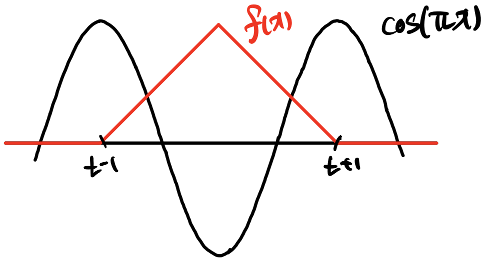
$$\therefore g(t) = \begin{cases} 0 & (t \le k-1) \\ \dfrac{\cos(\pi t)-1}{\pi^2} & (k-1 < t \le k) \\ \dfrac{3\cos(\pi t)+1}{\pi^2} & (k < t \le k+1) \\ \dfrac{4\cos(\pi t)}{\pi^2} & (k+1 < t \le k+7) \\ \dfrac{3\cos(\pi t)+1}{\pi^2} & (k+7 < t \le k+8) \\ \dfrac{\cos(\pi t)-1}{\pi^2} & (k+8 < t \le k+9) \\ 0 & (t > k+9) \end{cases}$$

sol1 과 동일

sol(3) 직관! + 가장 현실적 풀이

$f(x)$의 구간 $[k-1, k+1]$의 길이와 $\cos(\pi x)$의 주기가 같다.

그래프가 이렇지 않을까 추론:



: 최대

$\cos(\pi x) = -1$: $x = \pm 1, \pm 3, \pm 5, \cdots$

$\alpha_{1\sim5} = 5, 7, 9, 11, 13$ ($\because g(t)$의 구간 길이는 8)

$\therefore k=5, m=5$

$\int_0^1 (1-x)\cos(\pi x)dx = \frac{2}{\pi^2}$

$\therefore g(\alpha_1) = g(\alpha_5) = -\frac{2}{\pi^2}$

$g(\alpha_2) = g(\alpha_3) = g(\alpha_4) = -\frac{4}{\pi^2}$

$\therefore \sum g(\alpha_i) = -\frac{16}{\pi^2}$

$\therefore k - \pi^2 \sum g(\alpha_i) = 21$