함수 $f(x)=\ln\left(e^{x}+1\right)+2e^{x}$에 대하여 이차함수 $g(x)$와 실수 k는 다음 조건을 만족시킨다.

> 함수 $h(x)=|g(x)-f(x-k)|$는 $x=k$에서 최솟값 $g(k)$를 갖고, 닫힌 구간 $[k-1, k+1]$에서 최댓값 $2e+\ln\left(\frac{1+e}{\sqrt{2}}\right)$를 갖는다.

$g'\left(k-\frac{1}{2}\right)$의 값을 구하시오. (단, $\frac{5}{2}<e<3$이다.) [4점] 6

$h(k)=|g(k)-f(0)|=g(k)$

$\therefore f(0)=2g(k)=\ln 2+2 \quad (\because f(x)>0)$

$\therefore g(k)=\frac{\ln 2+2}{2}$: 최솟값

$f'(x)>0,\ f''(x)>0$: 증가함수, 아래로 볼록

$\longrightarrow$ if) $g(x)$가 아·볼 : $h(x)=0 \longrightarrow$ 모순 ($\because f, g$ 교점 존재)

$\therefore g(x)$는 위·볼!

$\therefore f(x-k)>g(x)$

$\therefore h(x)=f(x-k)-g(x)$

$h'(x)=f'(x-k)-g'(x)=0$

$\therefore g'(k)=\frac{5}{2} \quad \left(\because f'(x)=\frac{e^{x}}{e^{x}+1}+2e^{x}\right)$

$g(x)=ax^2+bx+c$, $g'(x)=2ax+b$

$g'(k)=2ak+b=\frac{5}{2}$

$$h(k-1)-h(k+1)=f(-1)-g(k-1)-f(1)+g(k+1)$$

$$=\frac{2}{e}-1-2e+\{4ak+b\}$$

$$=\underbrace{\frac{2}{e}}_{0.735\cdots}\underbrace{-1-2e+5}_{-1.436\cdots}<0$$

$\therefore$ 최댓값 : $h(k+1)$

$$h(k+1)=\ln(e+1)+2e-g(k+1)=2e+\ln(1+e)-\ln(\sqrt{2})$$

$$\therefore g(k+1)=\ln(\sqrt{2})=\frac{1}{2}\ln 2$$

$$g(k+1)-g(k)=2ak+b+a=-1$$

$$\therefore a=-\frac{7}{2}$$

$$\therefore g'\left(k-\frac{1}{2}\right)=2ak+b-a=6$$